A geometric approach to symmetric-key cryptanalysis

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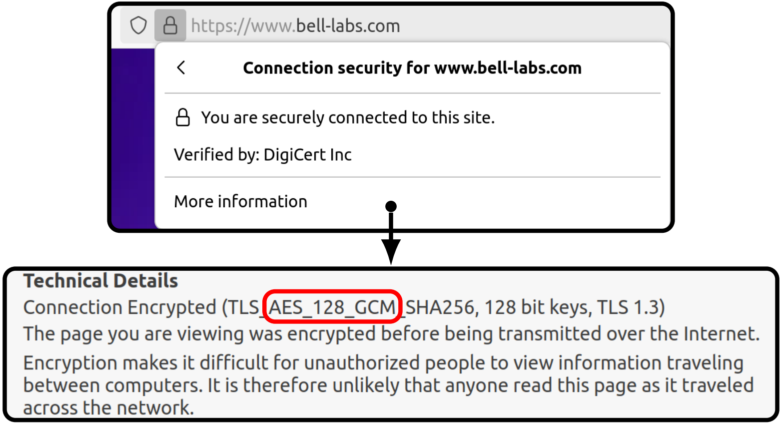
October 24, 2024

KU LEUVEN


https://www.bell-labs.com
Connection security for www.bell-labs.com
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Technical Details
Connection Encrypted (TLS_AES_128_GCM_SHA256, 128 bit keys, TLS 1.3)
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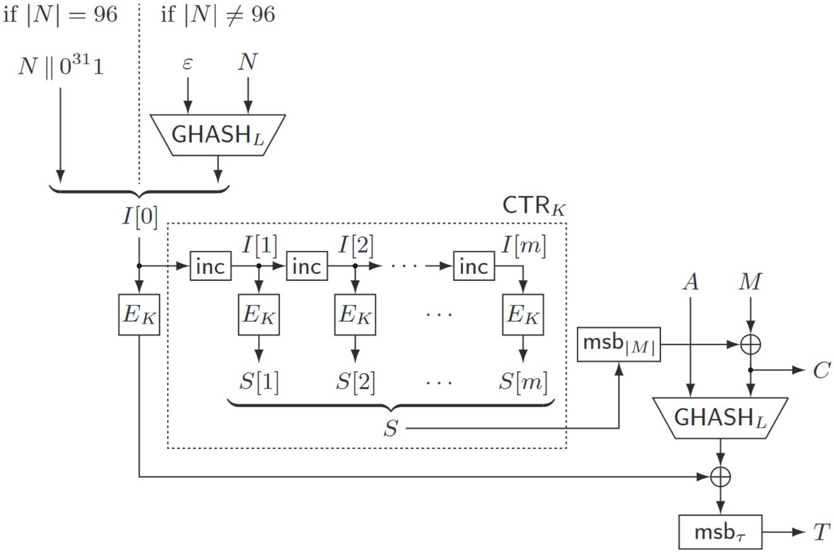
if $|N| = 96$
if $|N| \neq 96$
$N \,\|\, 0^{31}1$
ε
N
GHASH_L
$I[0]$
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$I[m]$
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$S[1]$
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A
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C
GHASH_L
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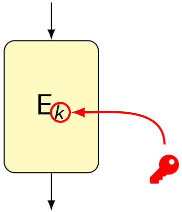
T

Block ciphers



e.g. AES-128

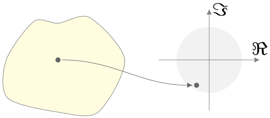
Cryptanalysis

Cryptanalysis

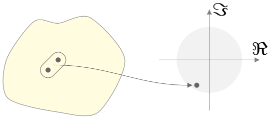
Overview

Geometric approach

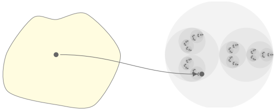
Linear cryptanalysis



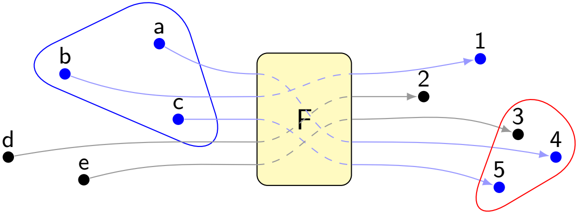
Differential cryptanalysis



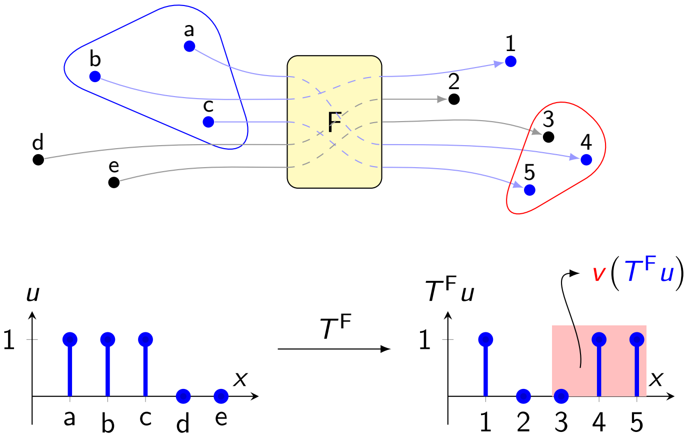
Integral cryptanalysis



Geometric approach

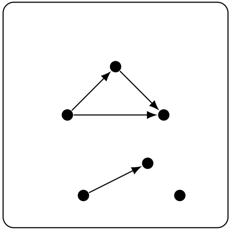


Geometric approach

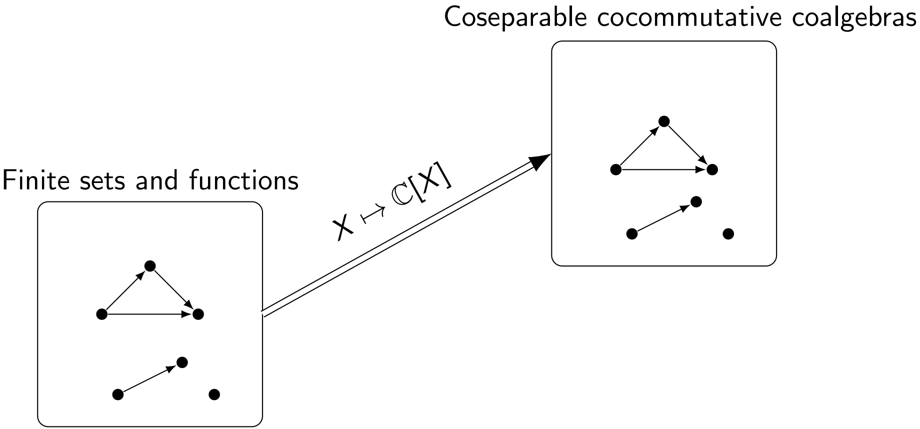


Geometric approach

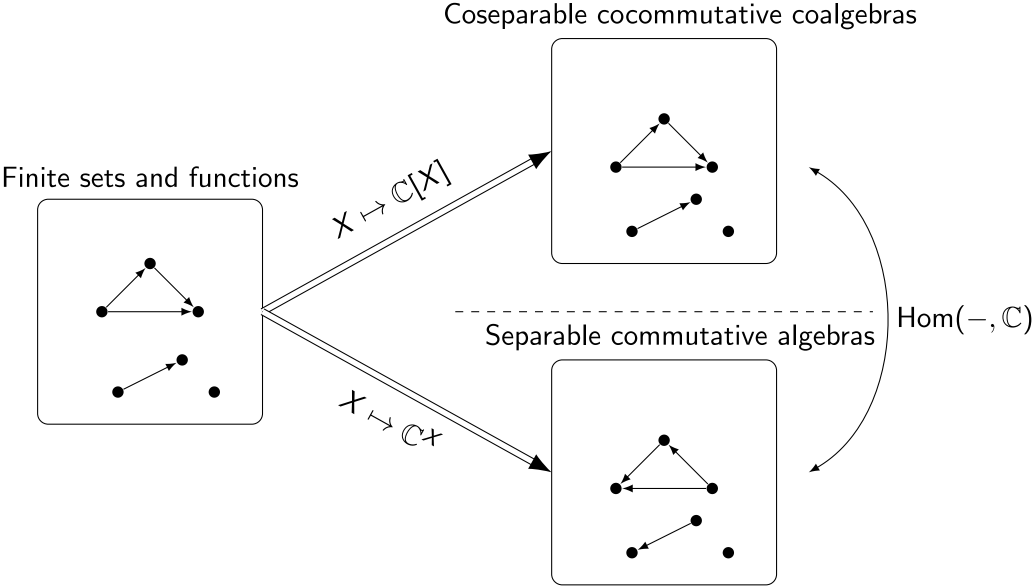
Finite sets and functions



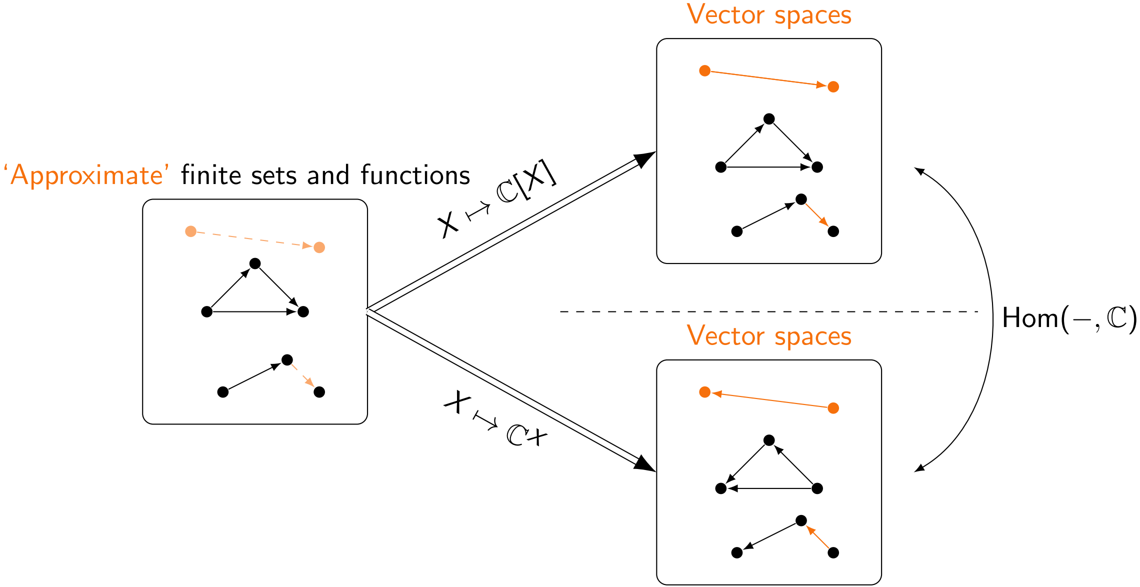
Geometric approach



Geometric approach

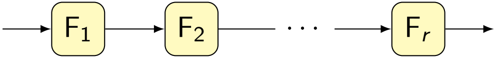


Geometric approach



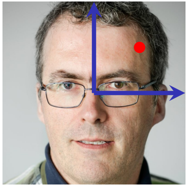
Iterated functions

- Evaluating $v(T^{\mathsf{F}} u)$ directly is not feasible for real ciphers
- Iterated structure of F:

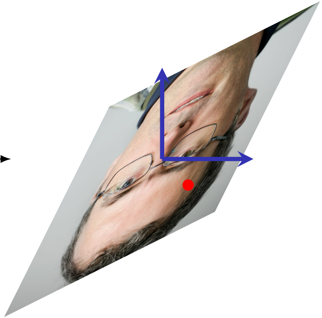


$$T^{\mathsf{F}} = T^{\mathsf{F}_r} \cdots T^{\mathsf{F}_2} T^{\mathsf{F}_1}$$

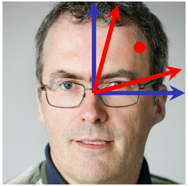
Change-of-basis



$$\begin{bmatrix} 1.15 & -0.58 \\ 0.58 & -1.15 \end{bmatrix}$$



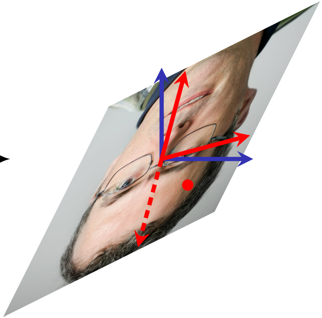
Change-of-basis



$$\begin{bmatrix} 1.15 & -0.58 \\ 0.58 & -1.15 \end{bmatrix}$$

$$\longrightarrow$$

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$



One-dimensional trails

$$T^{\mathsf{F}} \xleftrightarrow{\text{Change of basis}} B^{\mathsf{F}}$$

$$T^{\mathsf{F}} = T^{\mathsf{F}_r} \cdots T^{\mathsf{F}_2} T^{\mathsf{F}_1} \qquad B^{\mathsf{F}} = B^{\mathsf{F}_r} \cdots B^{\mathsf{F}_2} B^{\mathsf{F}_1}$$

- With the right change of basis, this makes it easier to estimate $v(T^{\mathsf{F}} u) = \widehat{v}(B^{\mathsf{F}} \widehat{u})$

One-dimensional trails

$$T^{\mathsf{F}} \longleftarrow \text{Change of basis} \longrightarrow B^{\mathsf{F}}$$

$$T^{\mathsf{F}} = T^{\mathsf{F}_r} \dots T^{\mathsf{F}_2} T^{\mathsf{F}_1} \qquad\qquad B^{\mathsf{F}} = B^{\mathsf{F}_r} \dots B^{\mathsf{F}_2} B^{\mathsf{F}_1}$$

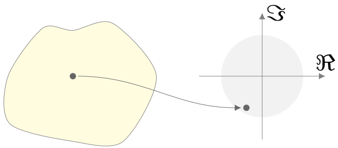
- With the right change of basis, this makes it easier to estimate $v(T^{\mathsf{F}} u) = \widehat{v}(B^{\mathsf{F}} \widehat{u})$
- When $u = b_{\beta_1}$ and $v = b^{\beta_{r+1}}$ are basis vectors:

$$b^{\beta_{r+1}}\left(T^{\mathsf{F}} b_{\beta_1}\right) = B^{\mathsf{F}}_{\beta_{r+1},\beta_1} = \sum_{\beta_2,\dots,\beta_r} \underbrace{\prod_{i=1}^{r} B^{\mathsf{F}_i}_{\beta_{i+1},\beta_i}}$$

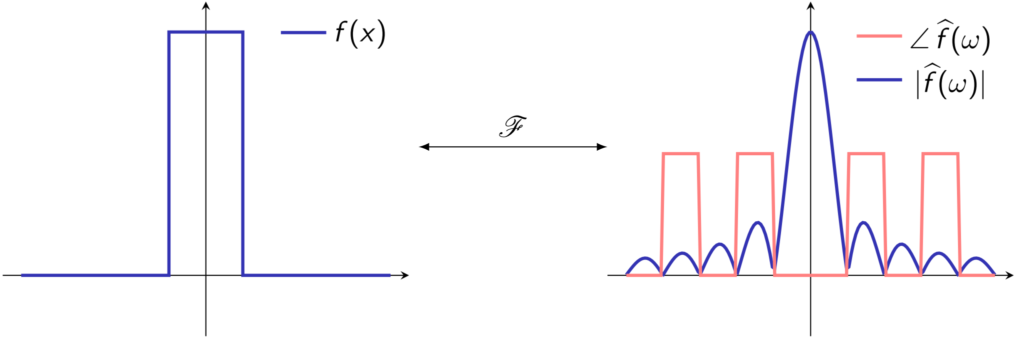
Trail correlation

- A sequence $(\beta_1, \dots, \beta_{r+1})$ of basis vector labels is a 'trail'

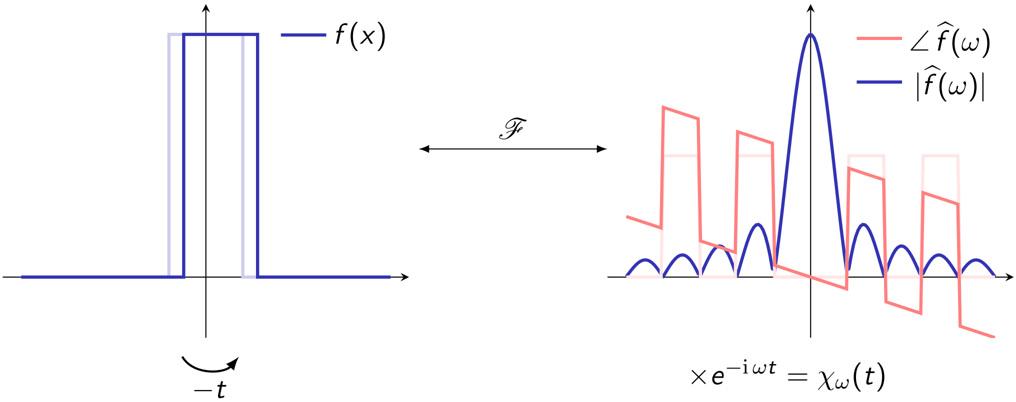
Linear cryptanalysis



Fourier transformation



Fourier transformation

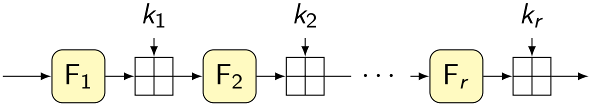


Fourier transformation diagonalizes translation

Geometric approach to linear cryptanalysis

[Asiacrypt 2021]

- Fourier transformation exists for any finite Abelian group (e.g. $\mathbb{Z}/N\mathbb{Z}$)



$$T^{\mathsf{F}} = {\color{red}T^{k_r}}\, T^{\mathsf{F}_r} \cdots {\color{red}T^{k_2}}\, T^{\mathsf{F}_2}\, {\color{red}T^{k_1}}\, T^{\mathsf{F}_1}$$

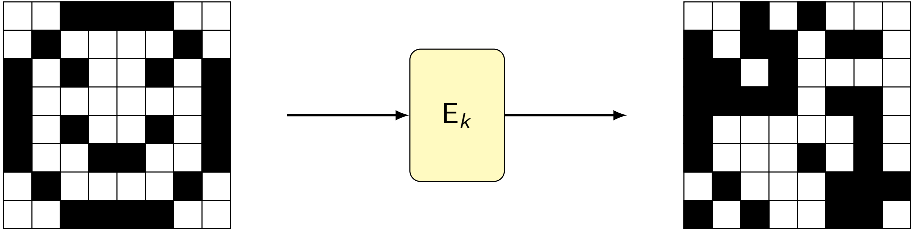
$$\Updownarrow \mathscr{F}$$

$$C^{\mathsf{F}} = {\color{pink}C^{k_r}}\, C^{\mathsf{F}_r} \cdots {\color{pink}C^{k_2}}\, C^{\mathsf{F}_2}\, {\color{pink}C^{k_1}}\, C^{\mathsf{F}_1}$$

Invariants

[Asiacrypt 2018, JoC 2020]

Example: Midori-64*

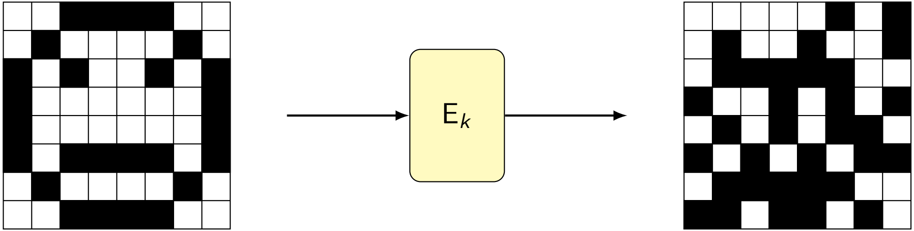


$2 + 3 + 4 + 4 \;= \text{odd}$ $\qquad$ $3 + 3 + 4 + 1 \;= \text{odd}$

[Asiacrypt 2018, JoC 2020]

Invariants

Example: Midori-64*



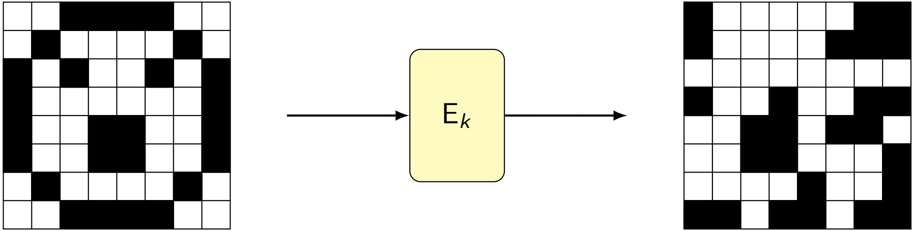
$2 + 3 + 4 + 4 = \text{odd}$
$2 + 3 + 4 + 4 = \text{odd}$

$3 + 3 + 4 + 1 = \text{odd}$
$5 + 5 + 5 + 4 = \text{odd}$

Invariants

[Asiacrypt 2018, JoC 2020]

Example: Midori-64*



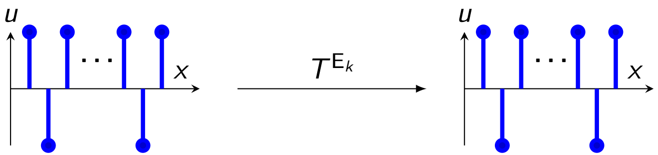
$2 + 3 + 4 + 4 = \text{odd}$
$2 + 3 + 4 + 4 = \text{odd}$
$2 + 4 + 3 + 4 = \text{odd}$

$3 + 3 + 4 + 1 = \text{odd}$
$5 + 5 + 5 + 4 = \text{odd}$
$1 + 4 + 2 + 6 = \text{odd}$

Invariants

Geometric approach

[Asiacrypt 2018, JoC 2020]

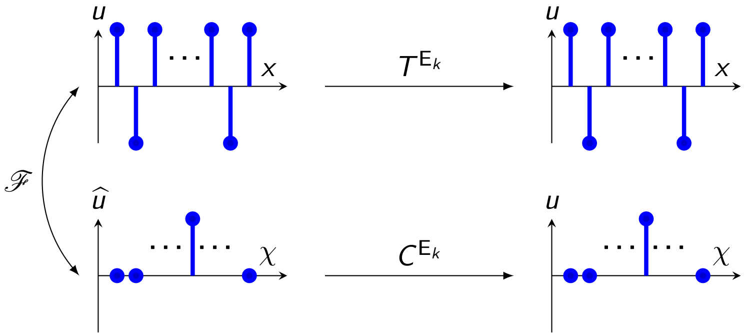


$$\begin{bmatrix} & & \\ & T^{E_k} & \\ & & \end{bmatrix} \begin{bmatrix} \\ u \\ \\ \end{bmatrix} = \begin{bmatrix} \\ u \\ \\ \end{bmatrix}$$

invariants are eigenvectors

Invariants

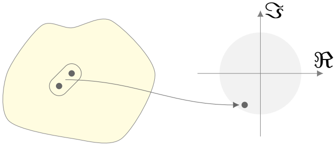
Geometric approach



$$\begin{bmatrix} & & \\ & C^{\mathsf{E}_k} & \\ & & \end{bmatrix} \begin{bmatrix} \\ \widehat{u} \\ \\ \end{bmatrix} = \begin{bmatrix} \\ \widehat{u} \\ \\ \end{bmatrix}$$

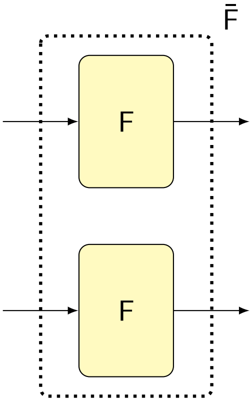
invariants are eigenvectors

Differential cryptanalysis



Pairs of values

- Assign weights (complex numbers) to all pairs of values



$$T^{\bar{\mathsf{F}}} = T^{\mathsf{F}} \otimes T^{\mathsf{F}}$$

Pairs of values

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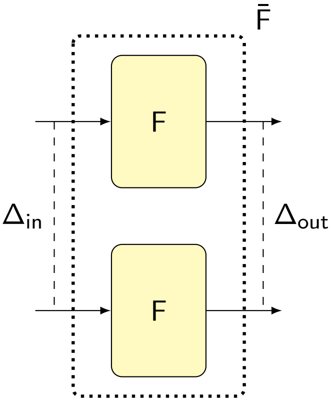


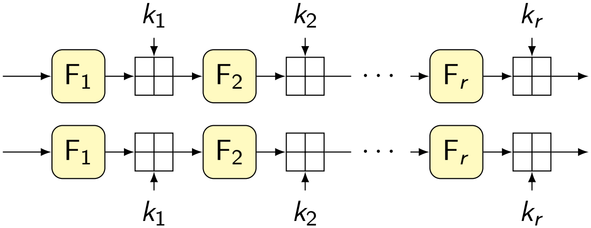
$$T^{\bar{\mathsf{F}}} = T^{\mathsf{F}} \otimes T^{\mathsf{F}}$$

Geometric approach to differential cryptanalysis

- Quasidifferential basis functions $(x, y) \mapsto \chi(x)\delta_a(y - x)$

Constant-difference pairs

Fourier basis



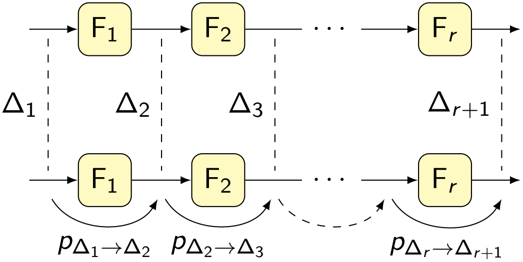
$$T^{\bar{\mathsf{F}}} = T^{\bar{k}_r} T^{\bar{\mathsf{F}}_r} \cdots T^{\bar{k}_2} T^{\bar{\mathsf{F}}_2} T^{\bar{k}_1} T^{\bar{\mathsf{F}}_1}$$

$$\Updownarrow \mathscr{Q}$$

$$D^{\mathsf{F}} = D^{k_r} D^{\mathsf{F}_r} \cdots D^{k_2} D^{\mathsf{F}_2} D^{k_1} D^{\mathsf{F}_1}$$

Reevaluating differential attacks

Independence assumptions

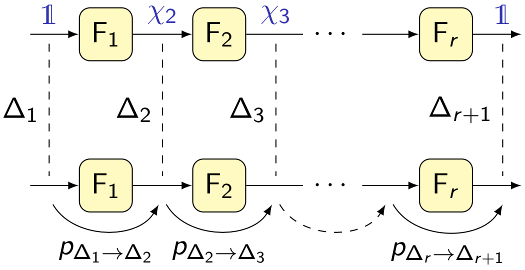


$$\text{probability} = \sum_{\Delta_2, \ldots, \Delta_r} p_{\Delta_1 \to \Delta_2} \times p_{\Delta_2 \to \Delta_3} \times \cdots \times p_{\Delta_r \to \Delta_{r+1}}$$

Reevaluating differential attacks

Independence assumptions

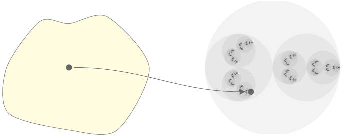
[Crypto 2022]



$$\text{probability} = \sum_{\Delta_2,\ldots,\Delta_r} \cancel{p_{\Delta_1 \to \Delta_2} \times p_{\Delta_2 \to \Delta_3} \times \cdots \times p_{\Delta_r \to \Delta_{r+1}}}$$

$$= \sum_{\substack{\Delta_2,\ldots,\Delta_r \\ \chi_2,\ldots,\chi_r}} D^{\mathsf{F}_1}_{(\chi_2,\Delta_2),(\mathbb{1},\Delta_1)} \times D^{\mathsf{F}_2}_{(\chi_3,\Delta_3),(\chi_2,\Delta_2)} \times \cdots \times D^{\mathsf{F}_r}_{(\mathbb{1},\Delta_{r+1}),(\chi_r,\Delta_r)}$$

Integral cryptanalysis



Geometric approach to integral cryptanalysis

- The Fourier transformation simplifies additions
  What about multiplications?

Geometric approach to integral cryptanalysis

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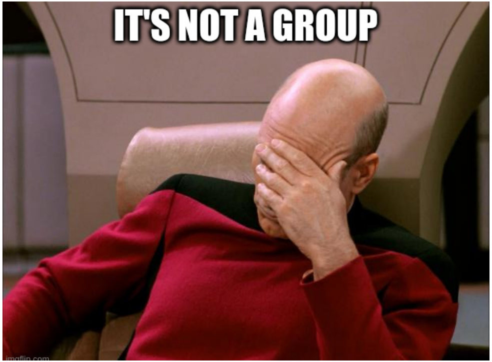
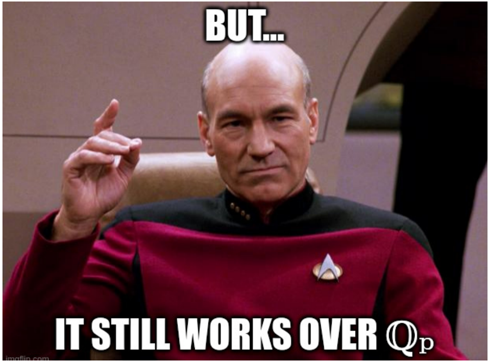
  What about multiplications?



Geometric approach to integral cryptanalysis

- The Fourier transformation simplifies additions
  What about multiplications?



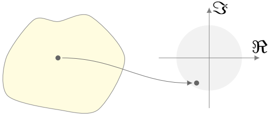
- Use weights in the p-adic numbers $\mathbb{Q}_p$

➜ 'Multiplicative' Fourier transformation that still preserves distances
… for some definiton of distance

Conclusions

Geometric approach

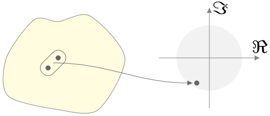
Linear cryptanalysis



Fourier basis

- Invariants of Midori & Mantis
- Attacks on FEA-$\{1,2\}$ & FF3-1
- Backdoored ciphers
- Side-channel countermeasures

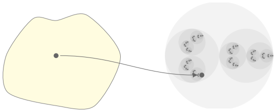
Differential cryptanalysis



Quasidifferential basis

- Re-evaluation of attacks (Speck, Rectangle, KNOT)
- Attacks on SM4 & GMiMC-crf
- Attacks on LowMC-M

Integral cryptanalysis



Ultrametric basis

- Ultrametric analysis of Present
- Attacks on Midori & Mantis
- Distinguishers for HadesMiMC

(other results: preimage attack on HadesMiMC, cryptanalysis of the Legendre PRF)